

Braneworld Dynamics with the BraneCode

[hep-th/0309001]

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Project with:

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Braneworlds

- ☞ Motivated by Superstring and Supergravity theories
- ☞ Applications to:

Particle Physics “Stringy” Models

- e.g. Hořava-Witten
- Interesting Phenomenology
- No Dynamics

Cosmology Toy Models

- e.g. Randall-Sundrum
- 4d-Gravity
- Trivial Dynamics
- Stabilization?

Braneworlds

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Particle Physics “Stringy” Models

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With the BraneCode

- Nontrivial Dynamics
- Geometric Inflation
- Dynamical Hierarchy
- Brane Collision

Cosmology

Toy Models

- e.g. Randall-Sundrum
- 4d-Gravity
- Trivial Dynamics
- Stabilization?

Action:

$$S = \int d^5x \sqrt{|g|} \left\{ \frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right\} - \sum_{i=1}^2 \int_{b_i} d^4\xi \sqrt{|\gamma|} \{ [K]_i + U_i(\phi) \}$$

Gauge:

$$ds^2 = e^{2B(y,t)}(dy^2 - dt^2) + e^{2A(y,t)}d\vec{x}^2$$

Comoving y - coordinate:

- Use gauge freedom to locate branes at $y = 0$ and $y = 1$
- Distance between branes: $D(t) = \int_0^1 e^{B(t,y)} dy$
- Proper time on either branes: $d\tau = e^B|_{y=0,1} dt$
- Induced 4d - Hubble parameter: $h = e^{-B}|_{y=0,1} \dot{A}$

E.O.M.:

$$\ddot{A} - A'' + 3\dot{A}^2 - 3A'^2 = \frac{2}{3} e^{2B} V$$

$$\ddot{B} - B'' - 3\dot{A}^2 + 3A'^2 + \frac{\dot{\phi}^2}{2} - \frac{\phi'^2}{2} = -\frac{1}{3} e^{2B} V$$

$$\ddot{\phi} - \phi'' + 3\dot{A}\dot{\phi} - 3A'\phi' = -e^{2B} \frac{dV}{d\phi}$$

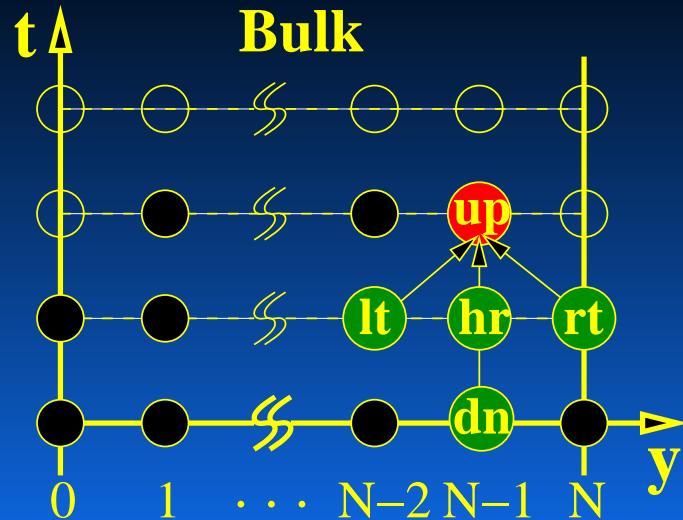
Two Constraint Equations

- Imposing initial conditions
- Check accuracy of numerical integration

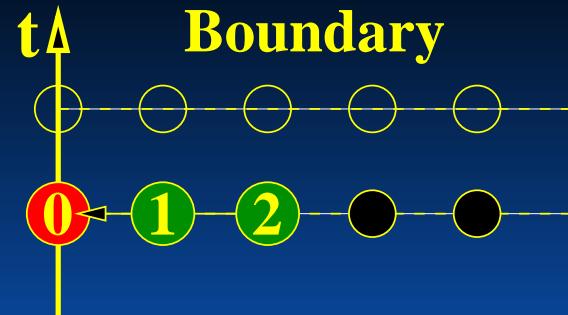
Boundary Conditions:

$$A'|_{y=0,1} = B'|_{y=0,1} = \mp \frac{1}{6} e^B U|_{y=0,1}$$

$$\phi'|_{y=0,1} = \pm \frac{1}{2} e^B \left. \frac{dU}{d\phi} \right|_{y=0,1}$$



Bulk Equations



Boundary Conditions

- ☞ Initial values given on first two grid lines
- ☞ Time evolution of $N - 1$ grid sites
- ☞ Determine field values at the position of the branes
- ☞ Second order accurate differencing scheme
 - Fields: $A(t, y)$, $B(t, y)$, $\phi(t, y)$
- ☞ Output:
 - Invariants: R , $C^{MNO}P C_{MNO}P$
 - Induced quantities: h_i , τ_i

Metric:

$$ds^2 = e^{2B(y,t)}(dy^2 - dt^2) + e^{2A(y,t)}d\vec{x}^2$$

Gauge transformation:

Preserve position of branes at

$$t + y \mapsto f(t + y)$$

- $y = 0: \Rightarrow f(x) = g(x)$

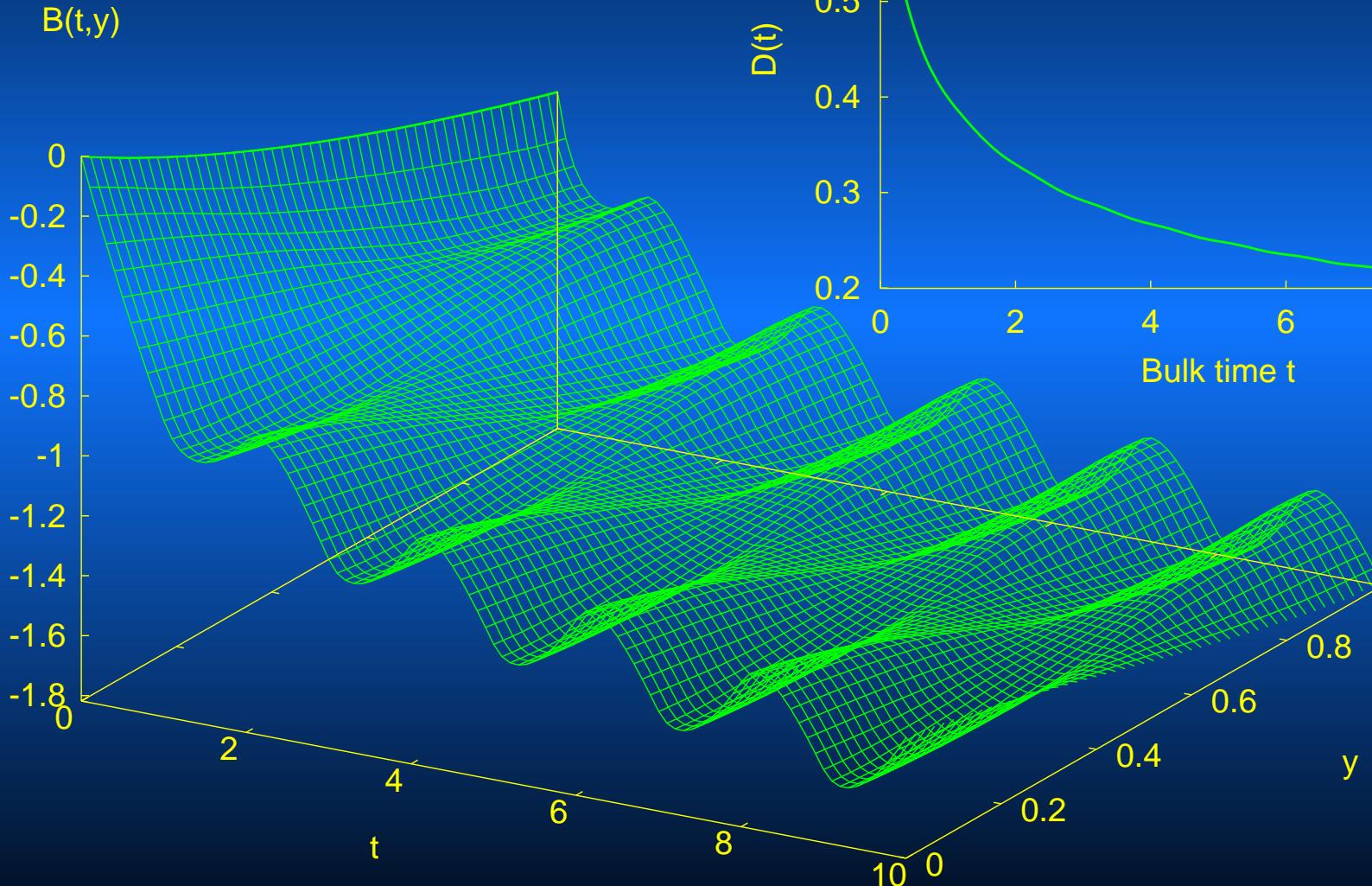
$$t - y \mapsto g(t - y)$$

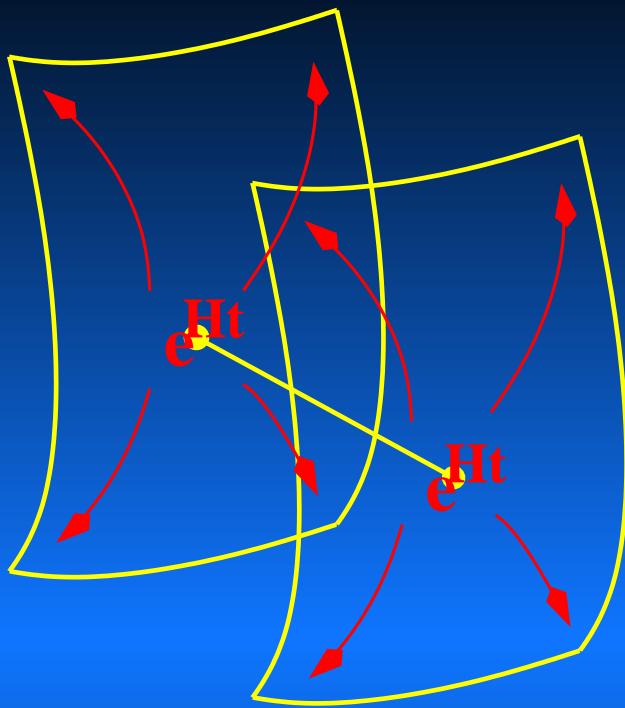
- $y = 1: \Rightarrow f'(x) = f'(x + 2)$

Residual gauge modes:

$$\tilde{y} = y + \sum_{n=1}^{\infty} \left\{ -a_n \sin(n\pi t) \sin(n\pi y) + b_n \sin(n\pi y) \cos(n\pi t) \right\}$$

$$\tilde{t} = t_0 + t + \sum_{n=1}^{\infty} \left\{ a_n \cos(n\pi t) \cos(n\pi y) + b_n \sin(n\pi t) \cos(n\pi y) \right\}$$





$$\begin{aligned}
 ds^2 &= e^{2B(y)} \{ dy^2 - dt^2 + e^{2Ht} d\vec{x}^2 \} \\
 \phi &= \phi(y) \\
 V &= \frac{1}{2} m^2 \phi^2 + \Lambda \\
 U_i &= \zeta_i (\phi - \nu_i)^2 + \lambda_i
 \end{aligned}$$

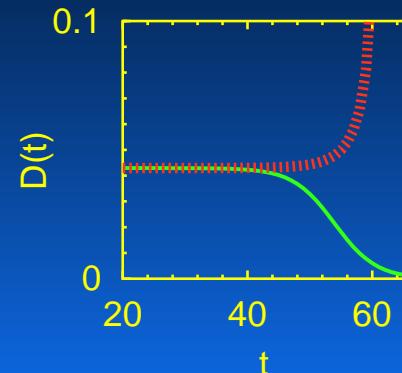
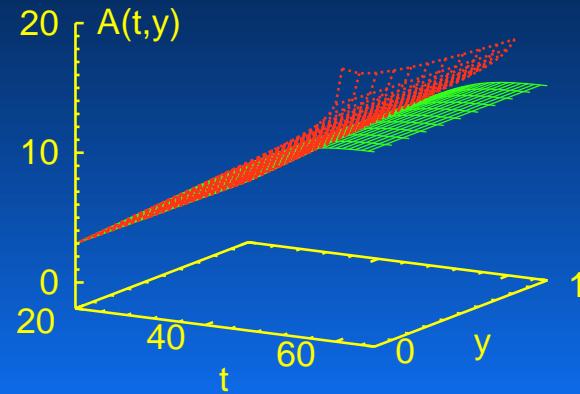
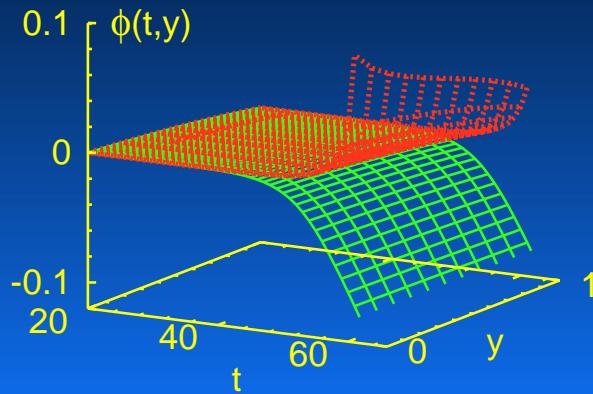
☞ Controllable initial conditions:

- $A(t, y) = B(y) + Ht$
- $B(t, y) = B(y)$
- $\phi(t, y) = \phi(y)$

☞ Goldberger-Wise potentials

☞ $a = e^{Ht}, D = \text{const}$

Stable solution ?

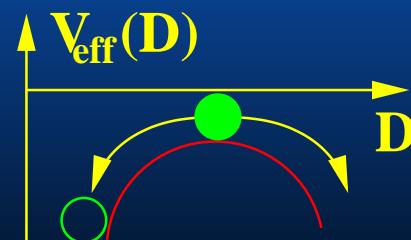
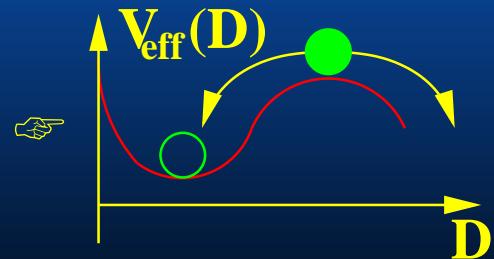


Physical Instability !

- Analysis of perturbations around stationary background

$$m_{\text{eff}}^2 \leq -4H^2 + m_0^2(\phi')$$

[A. Frolov, L. Kofman '03]



Given:

$$V(\phi), U_i(\phi)$$

Number of stationary solutions ?

- ☞ Nonlinear boundary value problem
- ☞ Two solutions possible

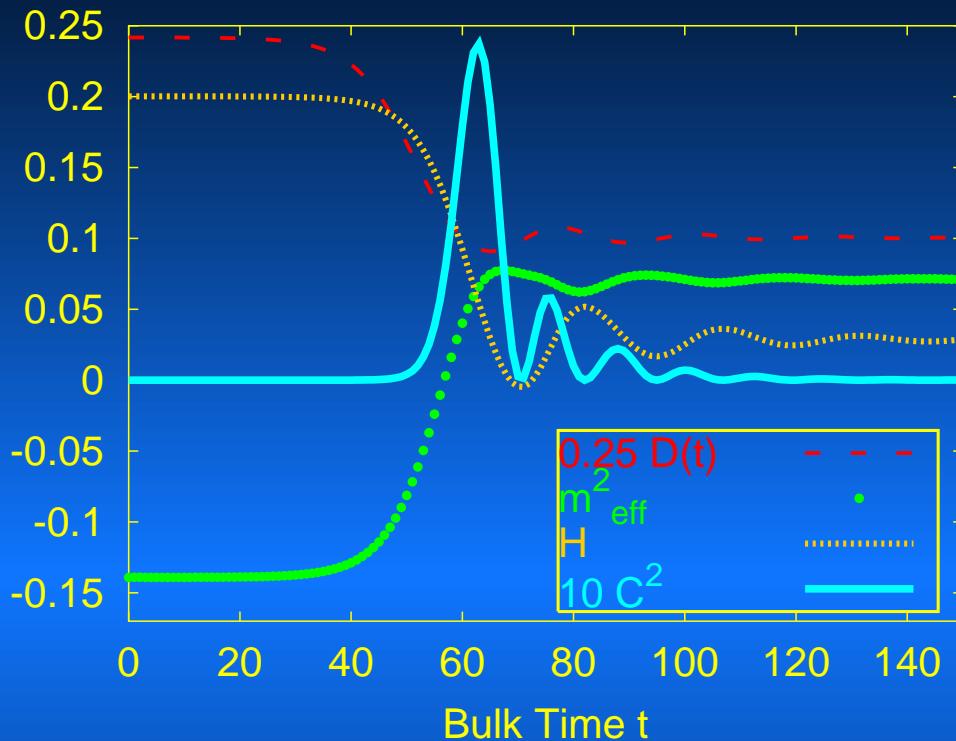
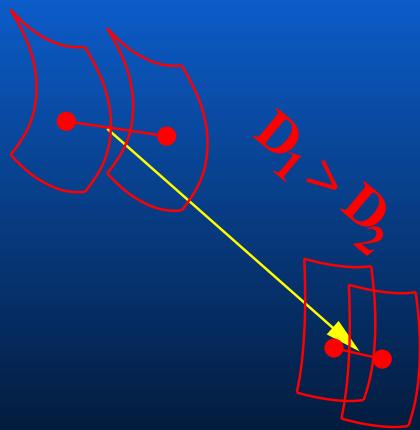
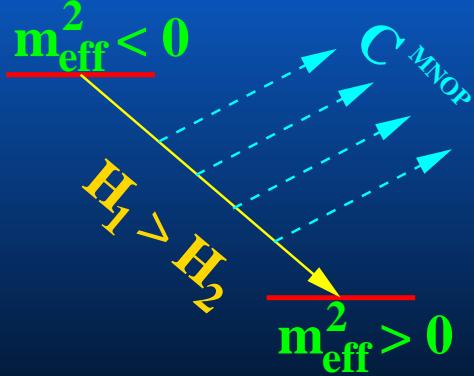
Transition ?

Given: $V(\phi), U_i(\phi)$

Number of stationary solutions ?

- ☞ Nonlinear boundary value problem
- ☞ Two solutions possible

Transition ?



- ☞ Nonlinear reconfiguration
- ☞ Transition towards flatter branes

Phasespace

Time independent problem:

System of first order ODEs of variables

$$\{\phi, e^{-B} \phi', e^{-B} B'\}$$

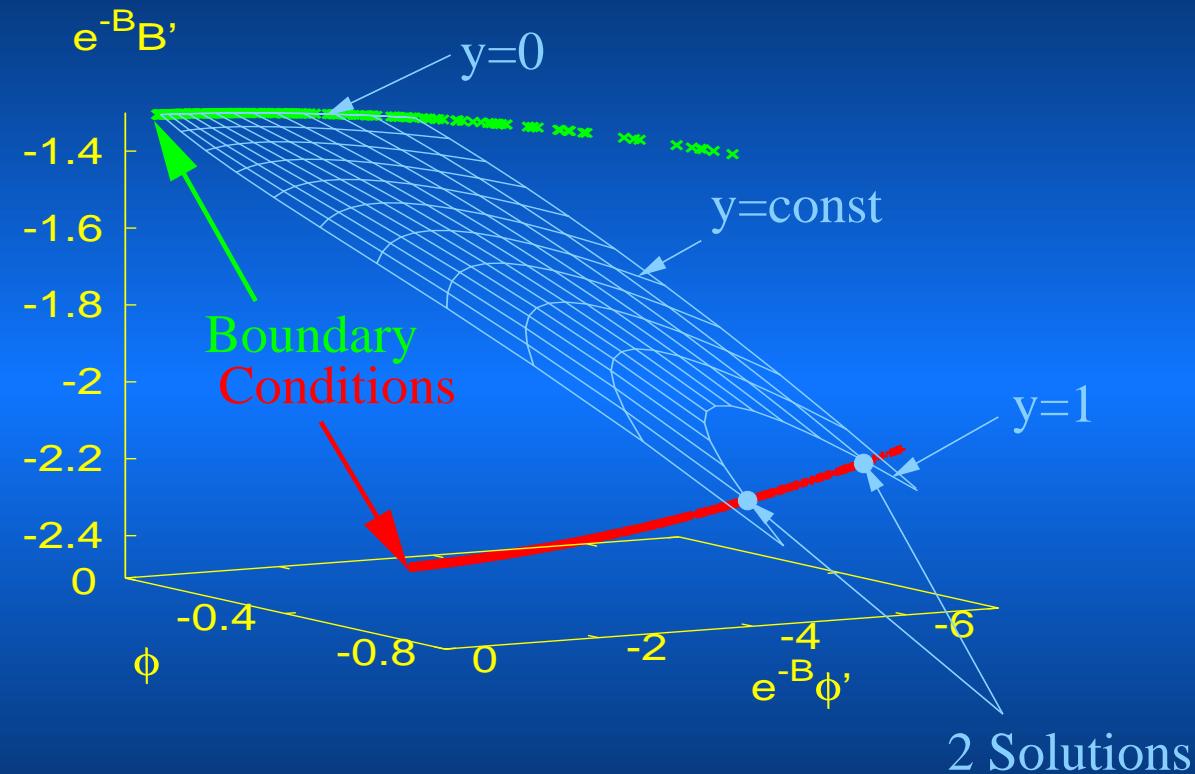
Solutions represented by trajectories

Boundary value problem

Boundary conditions:

$$e^{-B} B' = \mp \frac{1}{6} U(\phi)$$

$$e^{-B} \phi' = \pm \frac{1}{2} U'(\phi)$$



$$U_i = \zeta_i (\phi - \nu_i)^2 + \lambda_i$$

Moduli Approximation?

- Radion treated as light effective

4d - scalar field

- $\psi(\tau) \propto \ln\left(\frac{\tau}{\tau_c}\right)$

- $a(\tau) \propto \tau^{1/3}$

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Universal Attractor

- ☞ Solutions y - independent
- ☞ Potentials unimportant: $e^{2B} V \rightarrow 0$
- ☞ Analytic solution: 5D - Kasner-like solution + Scalar field

$$ds^2 = -d\tau^2 + \tau^{2p_y} dy^2 + \sum_{i=1}^3 \tau^{2p_i} dx_i^2$$

$$\phi = q \ln \tau$$

$$1 = p_y + p_1 + p_2 + p_3$$

$$1 - q^2 = p_y^2 + p_1^2 + p_2^2 + p_3^2$$

[V.Belinskiy, I.Khalatnikov '73]

Moduli Approximation?

- Radiation treated as light effective
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Strong 5d Gravity Regime

- ☞ $p_1 = p_2 = p_3$ (3-branes)
- ☞ $p_1 < \frac{1}{3}$
- ☞ Induced metric:

$$ds^2 = -d\tau^2 + (\tau_c - \tau)^{2p_1} d\vec{x}^2$$
- ☞ $D(t) \propto (\tau_c - \tau)^{p_y}$

Universal Attractor

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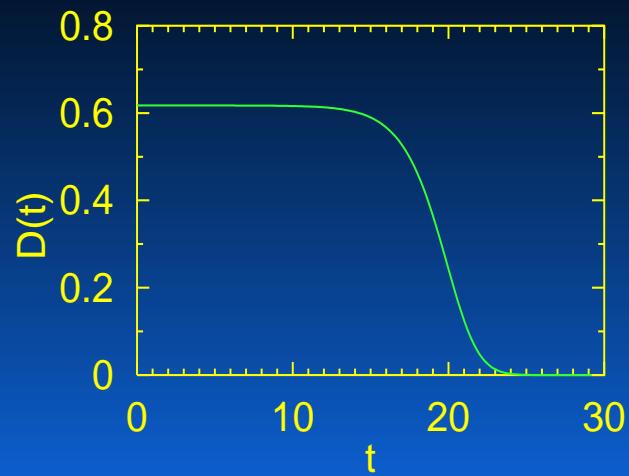
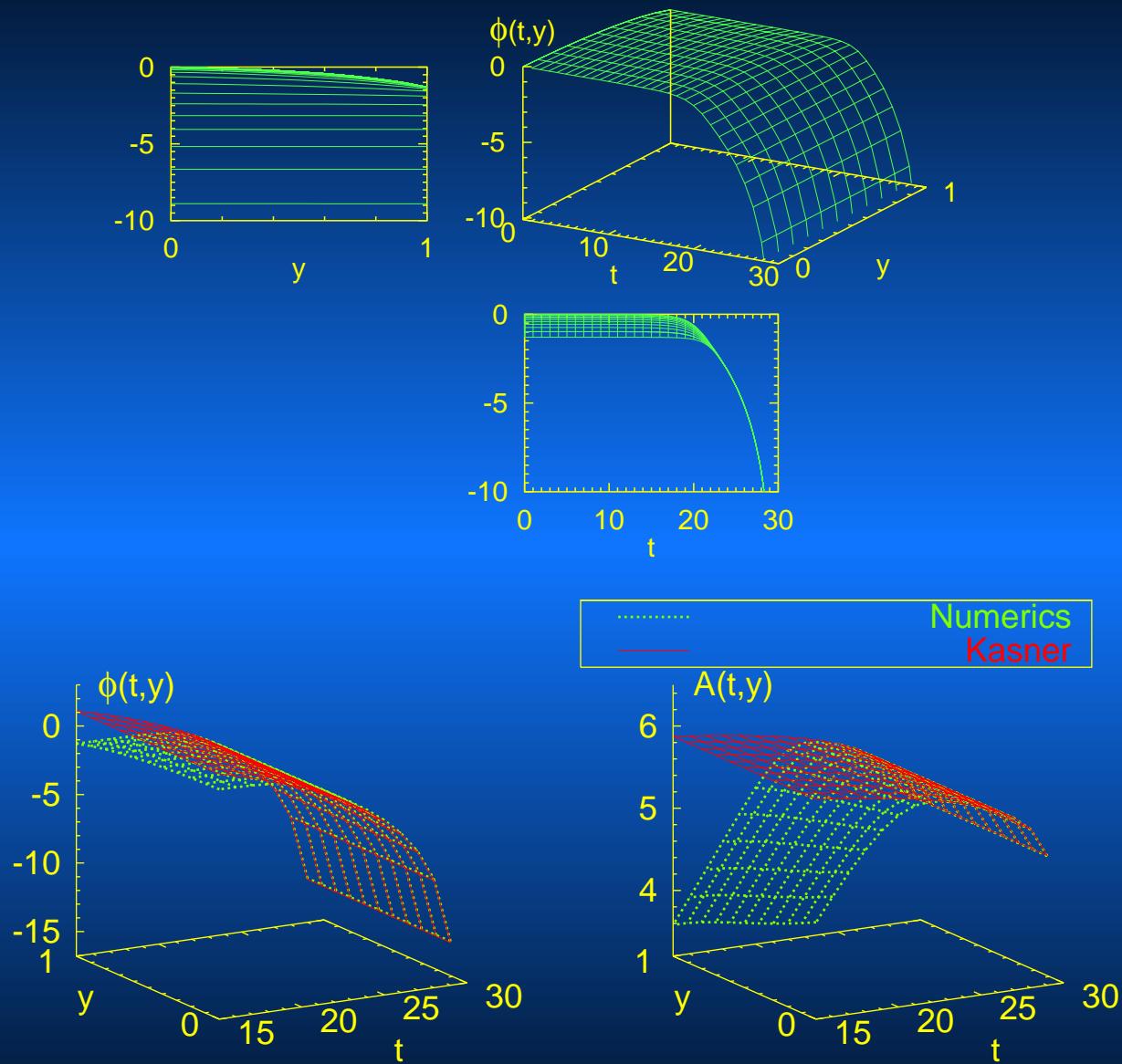
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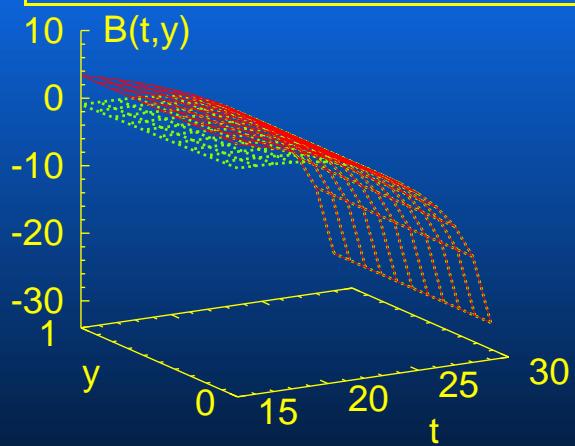
[V.Belinskiy, I.Khalatnikov '73]



$$A \propto \frac{1}{3} \ln(t_c - t)$$

$$B \propto \frac{p_y}{1-p_y} \ln(t_c - t)$$

$$\phi \propto \frac{q}{1-p_y} \ln(t_c - t)$$



- ☞ BraneCode for the Study of the Nonlinear System
- ☞ De-Sitter Branes Generically Unstable
- ☞ Dynamical Evolution Towards Flatter Branes
- ☞ Universal Attractor of Colliding Branes
- ☞ New Mechanism for the Generation of Perturbation

Perturbations:

$$ds^2 = e^{2B(y)} \left\{ \left(1 + 2\Phi(t, y, \vec{x}) \right) dy^2 + \left(1 + 2\Psi(t, y, \vec{x}) \right) \left[-dt^2 + e^{2Ht} d\vec{x}^2 \right] \right\}$$

$$y_i = \begin{cases} 0 + \xi_0(t, \vec{x}) \\ 1 + \xi_1(t, \vec{x}) \end{cases}$$

$$\delta\phi$$

E.O.M.:

$$\Psi = -\frac{1}{2}\Phi$$

$$\xi_i = 0$$

$$Y \equiv e^{2B}\Phi$$

$$Y = \sum_m Q_m(\vec{x}, t) Y_m(y) \quad {}^{(4)}\square Q_m = m^2 Q_m$$

$$\Rightarrow \boxed{-(g(y)Y'_m)' + f(y)Y_m = (m^2 + 4H^2)g(y)Y_m \quad Y_m'|_{y=0,1} = 0}$$

Mass Bound:

$$m^2 \leq -4H^2 + \frac{2}{3} \frac{\int_0^1 e^{-B} dy}{\int_0^1 \frac{e^{-B}}{\phi'^2} dy}$$